Chapter 19 - Technology

→ Now we will begin to derive the market supply curve from the optimization problems of individual firms.

→ Firms use inputs to make outputs, which they then sell

→ a firm's technology will define how it is able to turn inputs into outputs

Inputs and Outputs

→ Inputs are often called factors of production

→ Inputs can be land, equipment, raw materials, labor, etc.

→ we will generally divide things into capital and labor

→ capital = physical capital = machinery, buildings, computers, etc.

→ machines/structures built by other firms

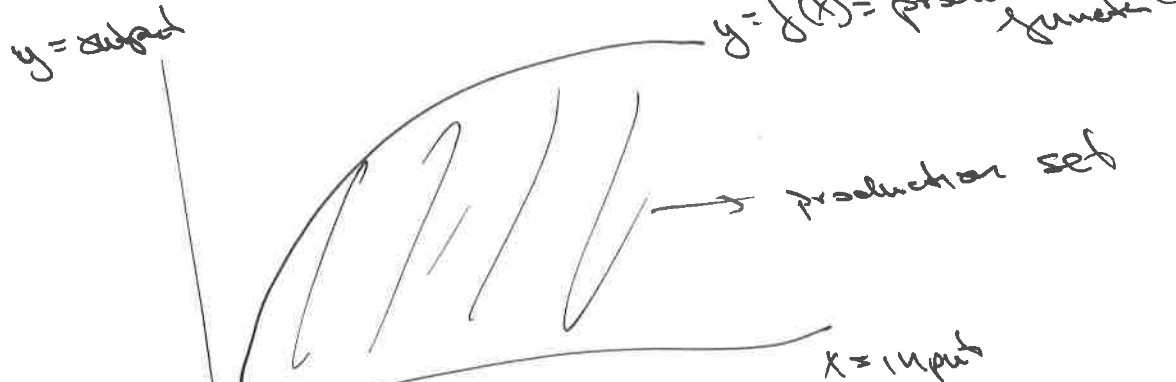
Technological Constraints

→ what is feasible to produce is given by the firm's production function

→ the production function, $f(x_1, x_2)$, tells us the maximum possible output that can be produced w/ x_1 and x_2

→ the production set are all the combinations of inputs and outputs that are feasible for a given technology (i.e. a given production function)

Graphically:



→ we can describe a technology w/ isoquants

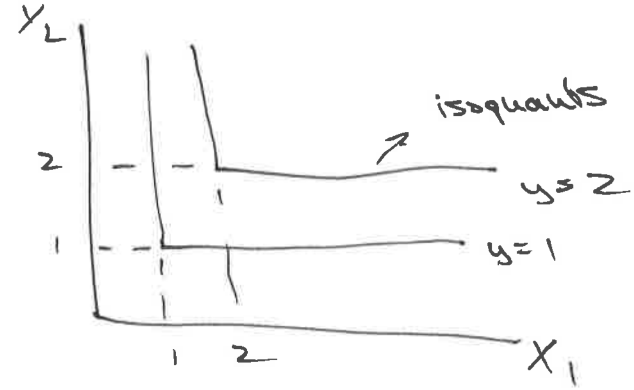
→ isoquants are curves that show all possible combinations of inputs that can be used to ~~achieve~~ just achieve a specific amount of output

→ Isoquants for some example technologies

1) Perfect complements (i.e. fixed proportions)

→ $f(x_1, x_2) = \min\{x_1, x_2\}$

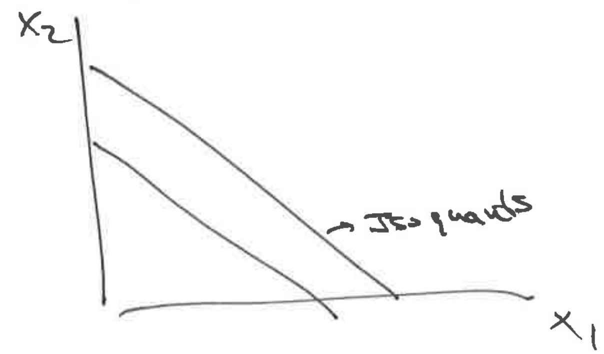
→ e.g. 1 boilermaker = 1 beer + 1 whiskey



2) Perfect substitutes

→ $f(x_1, x_2) = x_1 + x_2$

→ e.g. problem sets w/ pen or pencils



3) Cobb-Douglas

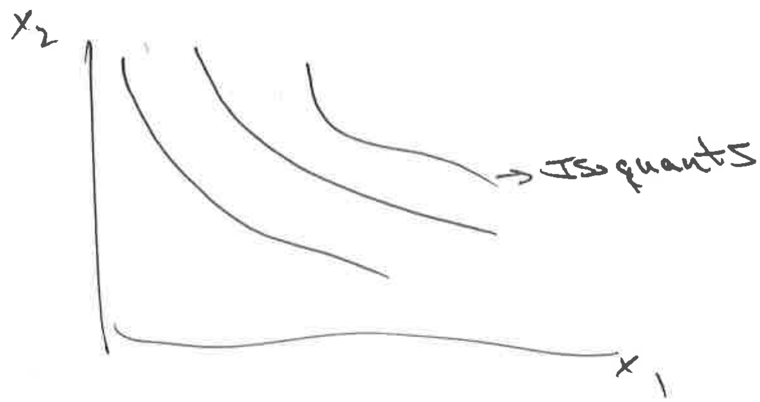
→ $y(x_1, x_2) = A x_1^a x_2^b$

→ A = scale of production (how much output if $x_1 = x_2 = 1$)

→ Note, we'll usually set $A = 1$

→ a, b = how much output responds to more x_1, x_2

→ can't set $a + b = 1$ ~~w/o changing~~ and retain same production function



Properties of technology

→ We will generally assume:

1) Technology is monotonic

→ more of at least one input and get at least as much output

→ related property of production functions: free disposal

→ firm can costlessly dispose of any input, so more inputs can't mean less output

2) Technology is convex

→ if (x_1, x_2) and (z_1, z_2) both produce y units of output, then weighted avg of (x_1, x_2) and (z_1, z_2) produce at least y units of output

→ The marginal product of a factor

→ the MP of a factor gives the rate of change in output for an increase in that factor:

$$MP \text{ of } x_1 = \frac{\partial y}{\partial x_1} = \lim_{\Delta x_1 \to 0} \frac{f(x_1 + \Delta x_1, x_2) - f(x_1, x_2)}{\Delta x_1}$$

$$\equiv MP_1(x_1, x_2)$$

The Technical Rate of Substitution

→ The technical rate of substitution gives the change in one factor for a change in another that keeps output constant

⇒ it's the slope of the isoquant

$$TRS(x_1, x_2) = \frac{\delta x_2}{\delta x_1}$$

$$\delta y = MP_1(x_1, x_2)\,\delta x_1 + MP_2(x_1, x_2)\,\delta x_2 = 0$$

$$\Rightarrow \frac{\delta x_2}{\delta x_1} = -\frac{MP_1(x_1, x_2)}{MP_2(x_1, x_2)}$$

$$\Rightarrow TRS(x_1, x_2) = -\frac{MP_1(x_1, x_2)}{MP_2(x_1, x_2)}$$

Properties of technology (cont'd)

3) Diminishing marginal products: as have more of input, the change in output for additional units of that input become smaller

$$\Rightarrow \frac{\partial MP_1(x_1, x_2)}{\partial x_1} < 0, \quad \frac{\partial MP_2(x_1, x_2)}{\partial x_2} < 0$$

↑ so prod func is concave

→ slope becomes flatter as ↑ x

4) Diminishing rate of technical substitution

→ Slope of isoquant decreases as ↑ x_1 and increases as ↑ x_2

→ basically, as have more and more of an input, you need to subs less of other to keep output constant

Returns to Scale

→ Returns to scale tells ~~use~~ us how output changes if we increase/decrease all inputs by a given percentage

→ constant returns to scale means output increases by the same factor as all inputs increase by:

$$f(zx_1, zx_2) = zy$$

→ decreasing returns to scale

$$f(zx_1, zx_2) < zy$$

→ increasing returns to scale:

$$f(zx_1, zx_2) > zy$$